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SIUE Drone Research
Weekly Update
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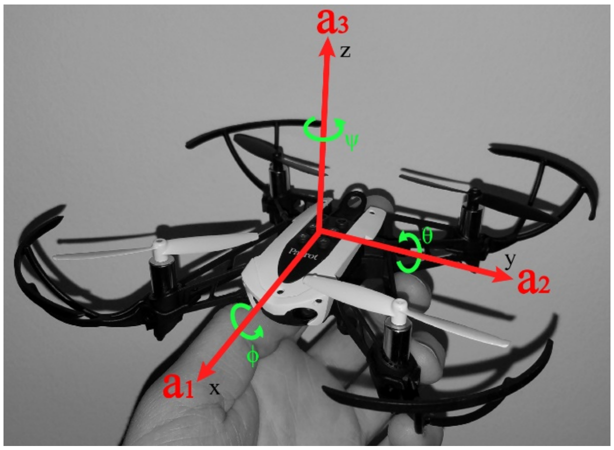
The Quadrotor Rigid Body – 6 Degrees of Freedom (6DOF)

We can represent the quadrotors as a rigid body, representing all rotations about the center of gravity. To do these we can setup two reference frames: the inertial reference frame which will be what the rigid body translates through- essentially a 3D spatial grid that we can assign coordinates and the body-fixed reference frame in which the origin is the center of gravity and all rotations are performed about the 3 axes.



Inertial Reference Frame:

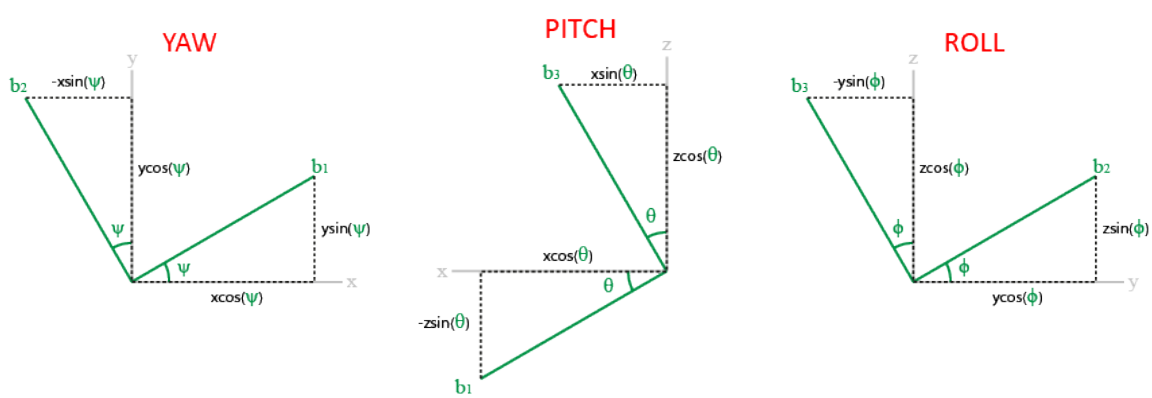
The image on the right shows the inertial reference frame of the quadcopter. The origin is at the assumed center of gravity and we can make rotations about it using Euler angles. There are three types of rotation:

Yaw (ψ) is rotation about z-axis (a_3)

Pitch (θ) is rotation about the y-axis (a_2)

Roll (ϕ) is rotation about the x-axis (a_1)

The 'b' vectors below are rotations from the initial y vectors. The illustrations represent how we calculate each rotation using trigonometry, represented in a 2D plane.



Now that we have each individual rotation it is easy to build the rotation matrices for each vector:

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\phi) & sin(\phi) \\ 0 & -sin(\phi) & cos(\phi) \end{bmatrix} \begin{bmatrix} cos(\theta) & 0 & -sin(\theta) \\ 0 & 1 & 0 \\ sin(\theta) & 0 & cos(\theta) \end{bmatrix} \begin{bmatrix} cos(\psi) & sin(\psi) & 0 \\ -sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Source: http://charlestytler.com/modeling-vehicle-dynamics-euler-angles/

And then combine them all into one:

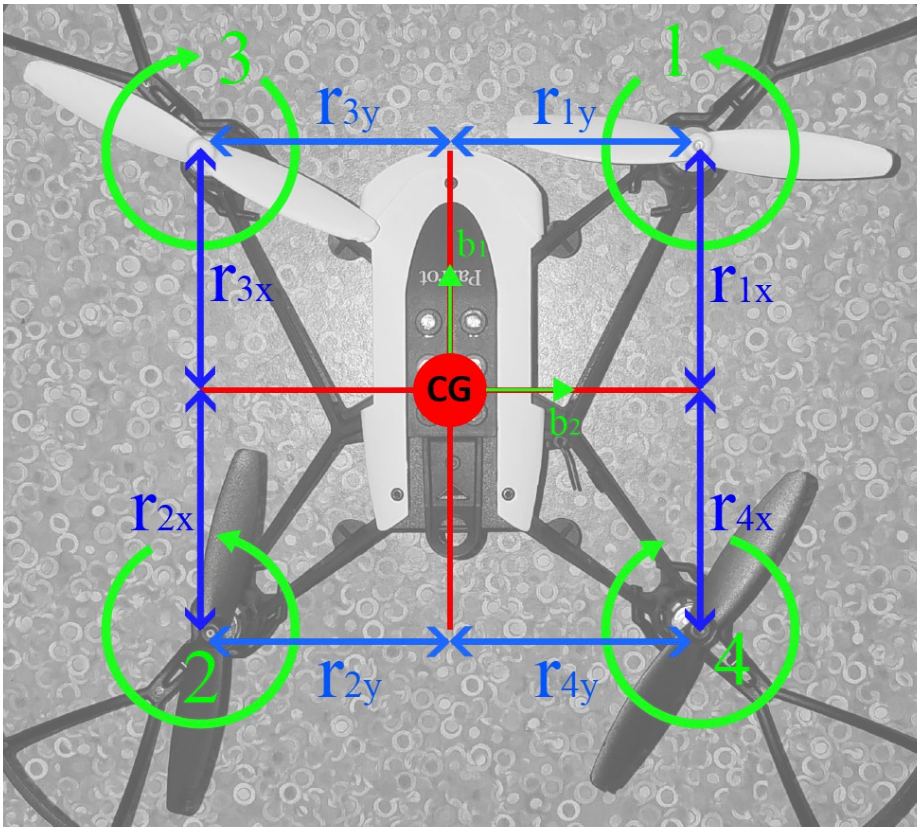
$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} cos(\theta)cos(\psi) & cos(\theta)sin(\psi) & -sin(\theta) \\ -cos(\phi)sin(\psi) + sin(\phi)sin(\theta)cos(\psi) & cos(\phi)cos(\psi) + sin(\phi)sin(\theta)sin(\psi) & sin(\phi)cos(\theta) \\ sin(\phi)sin(\psi) + cos(\phi)sin(\theta)cos(\psi) & -sin(\phi)cos(\psi) + cos(\phi)sin(\theta)sin(\psi) & cos(\phi)cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Source: http://charlestytler.com/modeling-vehicle-dynamics-euler-angles/

Kinematics

In terms of Moment…

Starting with moment we will be able to describe the forces about each axis and using the one about z we can construct kinematic equation for yaw.



The diagram to the right represents the thrust, rotation, and the distance between the props and the CG (also referred to as l in the previous report, when looked at on the xz plane). Each motor/prop is numbered, 1 and 2 have the same rotation while 3 and 4 have the opposing. We can use this information to find some equations for the moment of pitch (L), roll (M), and yaw (N). The moment matrix for these would look like:

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix}$$

Referencing the diagram above we can imagine that the z-axis (b_3) is coming out of the page. Now the thrust force for each motor can be assigned respectively as F_{T1} , F_{T2} , F_{T3} , and F_{T4} . Moment is calculated as force*distance, in our case distance from the cg is: $r_{\#axis}$.

$$Pitch\ (L) = F_1 r_{1y} - F_2 r_{2y} - F_3 r_{3y} + F_4 r_{4y}$$

$$Roll\ (M) = -F_1 r_{1x} + F_2 r_{2x} - F_3 r_{3x} + F_4 r_{4x}$$

The torque(τ) about the center of gravity is going to be what creates the yaw control, which we'll define as function of F, r_x, and r_y .

$$N = -T(F_1, r_{1x}, r_{1y}) - T(F_2, r_{2x}, r_{2y}) + T(F_3, r_{3x}, r_{3y}) + T(F_4, r_{4x}, r_{4y})$$

In order to solve this, we'll have to find the equation for Torque in respect to the three variables.