

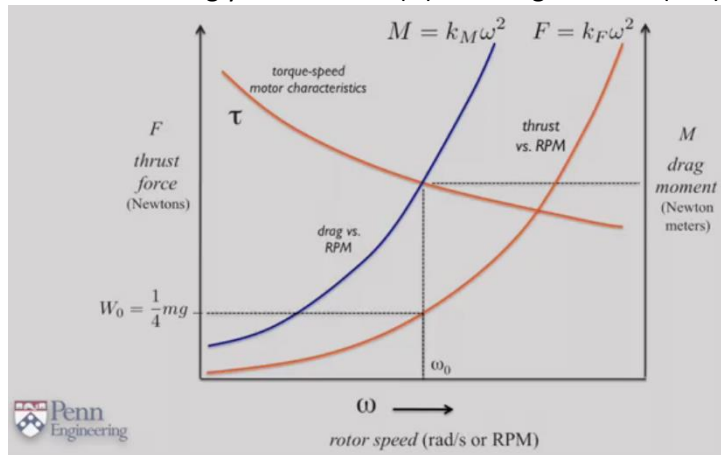
Summary

At last week’s meeting we discussed various topics, some short-term goals as well as long term possibilities for the research area. I set some goals for this past week to learn more about the kinematics of the quadcopter using online references, some of Diego’s finds, and centrally the Aerial Robotics class I am currently following on Coursera. I originally attempted to skip around the course and try to learn just the kinematic equations but found myself a little lost in the sea of variables that made little sense to me. I decided to take a step back and go through the course as it was meant to be presented, rushing the process was only causing confusion. As a result, I learned some interesting insight into control algorithms such as PID and PD, this should help when it comes to simulation as I’ll discuss further in my report. As of the present I have made it into the kinematics section and covered the first topic: thrust. This is the most basic of all the equations, but it’s helped me understand the variables and derivations more clearly. This week I also got the chance to run some pre-built simulations related to the PD control algorithm and they are great visual aids that I think will be very useful to the research.

Thrust and Moment vs RPM

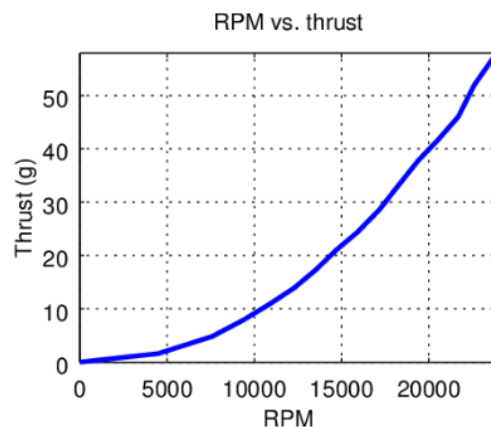
In selection of motors and rotors for the quadcopter there is an important thing to consider, that is the thrust and moment (drag) produced by each rotor. Interestingly when thrust (N) and drag moment (Nm)

are plotted against rotors speed (RPM or Rad/s) the resulting function is approximately quadratic, ideally. This relationship is illustrated in the graph shown to the right. The moment and thrust forces are on the y-axis and rotor speed (RPM) is on the x-axis. Torque can also be seen, but I have not yet addressed this relationship. From this graph the necessary speed of each motor can be calculated by $W_0 = \frac{1}{4}mg$, which is useful when defining necessary motor specifications.



Moment and thrust force formulas can also be seen on the graph as M and F. These formulas do not perfectly reflect the real world but that is where PID control will come into play, compensating for the imperfections. (Source: “Robotics: Aerial Robotics”)

In order to prove this relationship, I dove a little deeper and found some experimental results that prove a very similar relationship. To the right is the graph of the results, showing a very clear reflection of



the theory above. (Source: “Measuring Propeller RPM: Part 3”)

PD: Proportional Derivative Control

...is a control algorithm that uses the following equation to “smooth” the input values and create a smoothed output for the motors to follow.

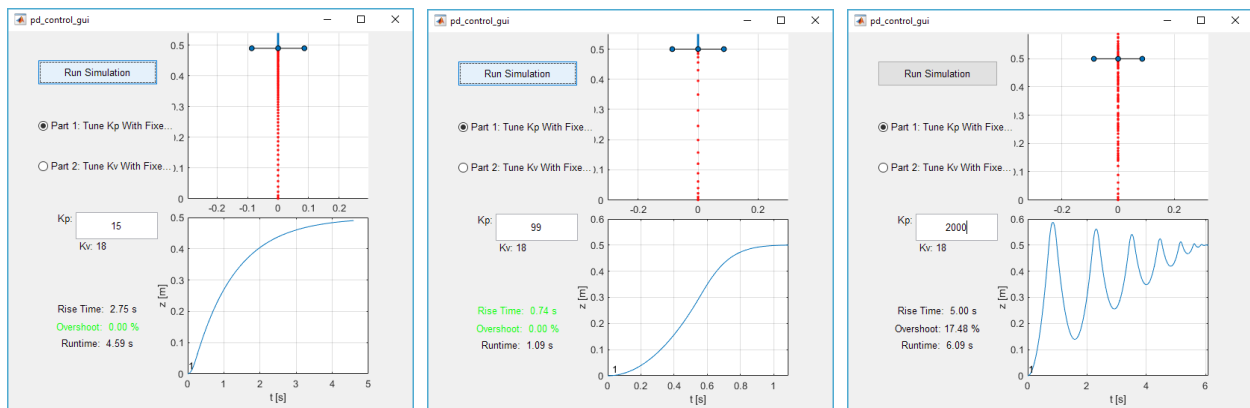
$$u(t) = \dot{x}^{des}(t) + K_r \dot{e}(t) + K_p e(t)$$

At my current point in research I do not fully understand what all these functions are responsible for individually, but I can break them easily into the two groups that give PD its name:

Proportional Control can be equated to the “springiness” of the response value, the higher the value the more “springy” response becomes. The higher the value, the more likely it will overshoot as well and take more time to approach the correct value, oscillating above and below it over time. This concept is easier to illustrate through a bit of visual simulation, which I included at the end of this PD section.

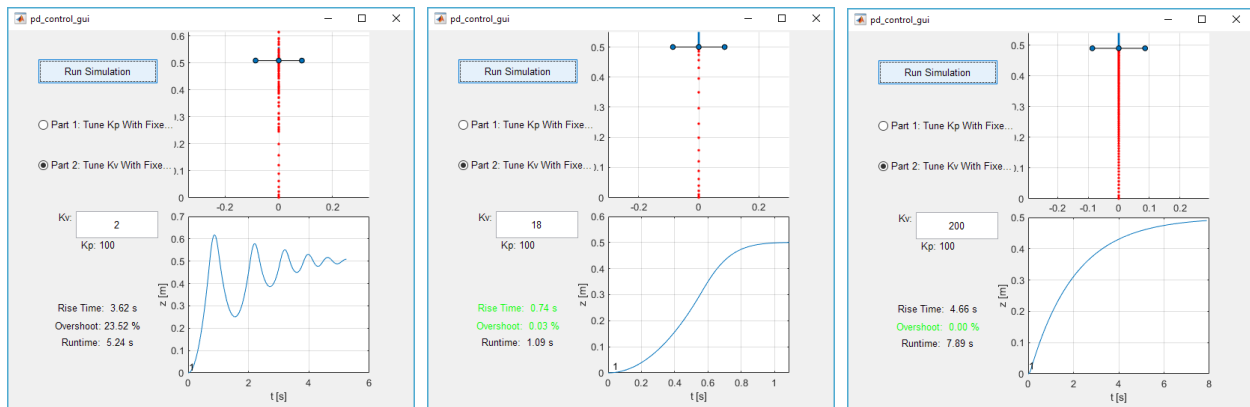
Derivative Control is described as a resistance, the higher the value- the longer it will take to approach the target value. This could also be understood as a density value, a fluid that is more viscous the higher the value. Once again- a much easier concept to illustrate visually.

Let’s look at some simulations in MATLAB:



This simulation code is provided as part of the Aerial Robotics course, I’ll make sure to add it to the shared drive. There are two input values available, K_p represents the proportional control and K_v the derivative control covered above. These first three tests shown above have a constant K_v and I am adjusting the K_p to illustrate the springiness. Respectively from left to right: the value is too low, the value is ideal, and the value is much too high. In the first trial the rise time is high, meaning the drone response (springiness) is slow. In the second the curve is nice and smooth, quick, and very little overshoot of the target value. The final trial shows the intense reaction when the control is too springy and overshoots the value by a lot and continues to compensate violently. In all the simulations the drone rising up can be mapped by the dots (representative of the slope of the graph) and in the center one there is a very nice concentration at either end of the timeline.

Okay- so we've looked at Proportional Control, now let's look at Derivative Control (The K_v Value), this time keeping K_p constant. We're going to use $K_v = 100$ as it was close to the best value found in the tests above.



We can see in these tests we can see the inverse of what we got before. At a lower value the response is more violent and higher is slower. This illustrates the idea of a resistive response where a higher value will slow the rise but reduce overshoot.

PID Control: Proportional Integral Derivative

...introduces a third variable into the control equation: the integral control. This type of control is used when unknown variables are introduced to the system such as wind resistance. For our indoor applications it can be used to balance out any remaining disturbances the drone may experience. As this is not the current focus of the research, I'll revisit PID later in the semester when it becomes necessary. For now, PD control will be useful for simulations that involve no unknown variables at the moment. I've included the general equation below for future reference:

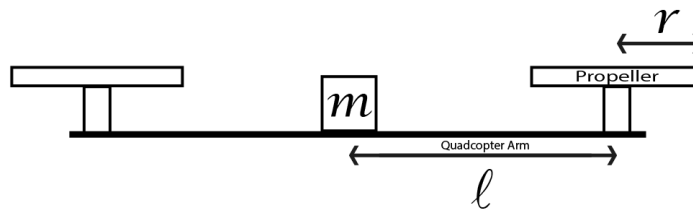
$$u(t) = \ddot{x}^{des}(t) + K_r \dot{e}(t) + K_p e(t) + K_i \int_0^t e(\tau) d\tau$$

Basic Equations

Mass, inertia: $m \sim l^3$

Thrust: $F \sim r^2 v^2$

$$F \sim l^2 v^2$$



Moment: $M \sim Fl$

$$M \sim r^2 v^2$$

Conclusion

Much of my research to date has been guided by the Aerial Robotics course and will most likely continue to be until I have finished. As I progress the kinematic equations will not only reveal themselves, but I hope with my background knowledge I'll be to understand and explain them fluently. Documenting my findings throughout will help with reference and please let me know if there is something I should be doing differently.

References

Kumar, Vijay. "Robotics: Aerial Robotics." *Coursera*, University of Pennsylvania, www.coursera.org/learn/robotics-flight/.

tobias. "Measuring Propeller RPM: Part 3." *Bitcraze*, 9 Feb. 2015, www.bitcraze.io/2015/02/measuring-propeller-rpm-part-3/.